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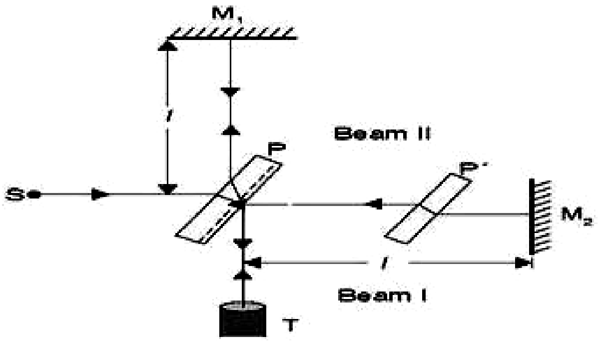
11.4 MICHELSON-MORLEY EXPERIMENT

In nineteenth century, scientists had assumed that a hypothetical medium called luminiferrous ether is required for the propagation of the light. It was considered that the ether exists uniformly in the space and it is at rest relative to the earth and other planets. The basic purpose of Michelson-Morley experiment was to confirm the existence of stationary ether. The existence of stationary ether (an absolute frame of reference) can be confirmed if we can measure the absolute velocity of earth with respect to the stationary ether.

The light beam moves to the mirror M_1 and the other moves to mirror M_2. These two beams are reflected back by these mirrors and again are recombined at plate P. Finally, they enter the telescope and produce interference. In the context of interference, this experiment has been discussed in detail in chapter 1.



In this experiment, the mirrors M_1 and M_2 are set such that $PM_1 = PM_2 = l$. According to Galilean transformation, the velocity of light in a frame moving with constant velocity v relative to stationary ether from P to M_2 is $(c - v)$ while from M_2 to P is $(c + v)$. If t_1 be the time taken by the transmitted beam from P to M_2 and back, then

$$t_1 = \frac{l}{c-v} + \frac{l}{c+v} \qquad \text{(i)}$$

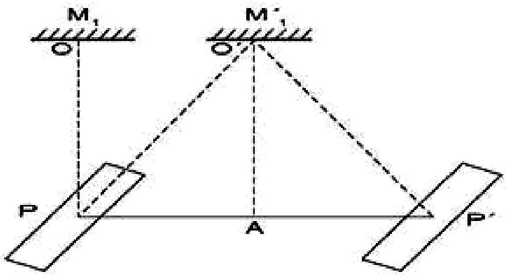
$$\text{or} \quad t_1 = \frac{2lc}{c^2 - v^2} = \frac{2l}{c}\left[\frac{1}{1 - v^2/c^2}\right]$$

$$\text{or} \quad t_1 = \frac{2l}{c}\left[1 - \frac{v^2}{c^2}\right]^{-1}$$

By using Binomial expansion theorem for the case of $v/c << 1$, we get

$$t_1 = \frac{2l}{c}\left[1 + \frac{v^2}{c^2}\right] \qquad \text{(ii)} \quad \text{[Neglecting higher order terms]}$$

The distance travelled by the light beam in time t_1 is given by



$$x_1 = t_1 \cdot c = 2l\left[1 + \frac{v^2}{c^2}\right] \qquad \text{(iii)}$$

If t_2' be the time taken by beam-II from P to M_1 and in the same time distance travelled by this beam is ct_2'. In this time t_2', the mirror M_1 shifts to M_1' and travels a horizontal distance vt_2'.

With the help of Fig. 11.4, we get $PO = l$, $PO' = ct_2'$ and $OO' = vt_2'$.

$$(PO')^2 = (PO)^2 + (OO')^2$$

$$(ct_2')^2 = (l)^2 + (vt_2')^2$$

$$\text{or} \quad (c^2 - v^2)t_2'^2 = l^2$$

$$t_2' = \left(\frac{l^2}{c^2 - v^2}\right)^{1/2}$$

$$\text{or} \quad t_2' = \frac{l}{c} \times \frac{1}{\sqrt{1 - v^2/c^2}} \tag{iv}$$

$$\text{Total time, } t_2 = 2t_2' = \frac{2l}{c} \times \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\text{or} \quad t_2 = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

By using Binomial expansion theorem, we obtain

$$t_2 = \frac{2l}{c}\left(1 + \frac{v^2}{2c^2}\right) \tag{v}$$

[Neglecting the higher order terms]

The distance travelled by the beam in time t_2 is given by

$$x_2 = c \cdot t_2$$

$$x_2 = 2l\left(1 + \frac{v^2}{2c^2}\right) \tag{vi}$$

With the help of Eqs. (iii) and (vi), we get the path difference,

$$\Delta x = x_1 - x_2 = 2l\left(1 + \frac{v^2}{c^2}\right) - 2l\left(1 + \frac{v^2}{2c^2}\right)$$

$$= 2l\left[1 + \frac{v^2}{c^2} - 1 - \frac{v^2}{2c^2}\right]$$

$$\therefore \quad \Delta x = \frac{lv^2}{c^2}$$

Because of the introduction of this path difference of the two beams, the interference pattern would be shifted as,

$$n = \frac{lv^2}{\lambda c^2}$$

[$\because$ path difference $= n\lambda$ for constructive interference]

where, λ is the wavelength of the light used.

If the apparatus is rotated through 90°, the reflected and transmitted beams get interchanged and the path difference of $\frac{lv^2}{c^2}$ will be produced in the opposite direction. This way the total path difference between the interfering beams becomes $\frac{2lv^2}{c^2}$ and the interference pattern would be shifted as,

$$n = \frac{2lv^2}{\lambda c^2} \tag{vii}$$

In the above Eq. (vii) on putting $l = 11$ meter, velocity of earth $v = 3 \times 10^4$ m/sec, $c = 3 \times 10^8$ m/sec and $\lambda = 5.5 \times 10^{-7}$ m, the expected fringe shift comes out to be

$$n = \frac{2lv^2}{\lambda c^2} = \frac{2 \times 11 \times (3 \times 10^4)^2}{5.5 \times 10^{-7} \times (3 \times 10^8)^2}$$

$$\text{or} \quad n = 0.4$$

Explanation of negative results of michelson-morley experiment

As per this hypothesis, all bodies moving with velocity v are contracted in the direction of motion by a factor $\sqrt{1-v^2/c^2}$. So if L_0 be the length of a body at rest with respect to ether and L be its length when the body is in motion with velocity v with respect to ether, then $L = L_0\sqrt{1-v^2/c^2}$. With this $t_1 = \frac{2L}{c}(1+v^2/c^2) = \frac{2L_0\sqrt{(1-v^2/c^2)}}{c}(1+v^2/c^2) = \frac{2L_0(1-v^2/c^2)}{c}(1+v^2/c^2)$. If we neglect the higher powers of v/c in view of $v < c$, then the time $t_1 = \frac{2L_0}{c}(1+v^2/2c^2) = t_2$. It means the time taken or path traversed by the reflected and transmitted beams is the same. So no shift of fringes is observed. However, this concept could not be applied for explaining the negative results of Michelson-Morley experiment when the two arms of the interferometer are not equal.